Math 126: Quiz 4
November 2, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic and trig/exponential functions only, i.e., no plotting and no calculus. PLEASE READ ALL DIRECTIONS CAREFULLY and JUSTIFY YOUR ANSWERS!

1. Solve the initial value problem $\frac{dy}{dx} = \frac{x}{y}$ subject to the condition $y(1) = 2$.

\[
\frac{dy}{dx} = \frac{x}{y} \implies y \, dy = x \, dx
\]

\[
\int y \, dy = \int x \, dx \\
\frac{y^2}{2} = \frac{x^2}{2} + C
\]

\[
y^2 = x^2 + C' \\
y = \sqrt{x^2 + C'}
\]

\[
y(1) = 2 \implies C' = 3
\]
2. Professor $B^2$ calls the cable company to complain that he can’t watch football. The company claims that the average wait time on hold is 5 minutes, giving a probability distribution function of $p(x) = 0.2e^{-0.2x}$.

(a) Find the probability that Prof $B^2$ gets his call answered between 2 and 4 minutes.

$$
\int_{2}^{4} 0.2 e^{-0.2t} \, dt = -e^{-0.2t} \bigg|_{2}^{4} \\
= -e^{-0.8} + e^{-4}
$$

$$
p = 0.221
$$

(b) Find the probability that Prof $B^2$ has to wait more than 15 minutes.

$$
\int_{15}^{\infty} 0.2 \, e^{-0.2t} \, dt = -e^{-0.2t} \bigg|_{15}^{\infty} \\
= e^{-0.2 \cdot \infty} - e^{-0.2 \cdot 15} = e^{-3}
$$

$$
p = 0.0497
$$

(c) Do most people have their calls answered before 5 minutes? Explain.

Most people have their calls answered before 5 minutes.

If 5 minutes is the average, it accounts for people who have to wait more than 10, 15, 20 minutes.
3. Find the length of the curve \( f(x) = 2x^{3/2} \) from \((1, 1)\) to \((4, 16)\).

\[
\text{Length: } \int_1^4 \sqrt{1 + f'(x)^2} \, dx = \int_1^4 \sqrt{1 + (3x^{1/2})^2} \, dx
\]

\[
= \int_1^4 \sqrt{1 + 9x} \, dx = \frac{1}{9} (1 + 9x)^{3/2} \bigg|_1^4
\]

\[
u = 1 + 9x \quad \Rightarrow \quad du = 9 \, dx \quad \Rightarrow \quad \frac{1}{9} \left[ (37^{3/2} - 10^{3/2}) \right]
\]

3. Set up, but don’t evaluate, the integral to compute the surface area generated when the curve in problem 2 is rotated about the \( x \)-axis.

\[
\text{SA} = \int_a^b 2\pi y \, ds
\]

\[
= \int_1^4 2\pi y \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

\[
= \int_1^4 2\pi y \left( 2x^{3/2} \right) \sqrt{1 + 9x} \, dx
\]

\[
\text{or} \quad \int_a^b 2\pi y \sqrt{1 + \left( \frac{1}{3} \left( \frac{1}{2} \right)^{-1/3} \right)^2} \, dy
\]
5. Hungary is one of the few countries in the world with a world with a decreasing population. Every year, 2 percent of the current population either dies off or leaves, while only 150,000 new people are born or move there. Hungary's current population is 10,000,000. Set up a differential equation to determine the population as a function of time. When will the population dip below 9,000,000? What is the projected long term population?

\[ \frac{dp}{dt} = -0.02p + 150,000 \]

\[ \frac{dp}{dt} = -0.02 \ (p \ - \ 7500) \]

\[ \frac{dp}{p-7500} = -0.02 \ dt \]

\[ \ln |p-7500| = -0.02t + C \]

\[ p = 7500 + Be^{-0.02t} \]

\[ B = \frac{7500 + 9000}{2} = 8250 \]

\[ 9000 = 7500 + 2500 e^{-0.02t} \]

\[ \ln \left( \frac{1500}{2500} \right) = \frac{25.57 \text{ years from now}}{-0.02} \]

\[ \text{long term pop} \rightarrow 7.5 \text{ million} \]

6. (Bonus) Find at least two functions \( y \) which satisfy the second order differential equation \( \frac{d^2y}{dx^2} = -y \). (This denotes the second derivative.)

Think Trig Functions!

\[ y = \cos x \]

\[ y = \sin x \]