Assignments for Math 244 for Spring 2015 (part 4)

for Tuesday, April 21

1. There is no particular assignment. We will be starting Chapter 6 on this day. The prerequisite knowledge that is assumed here consists primarily of an understanding of improper integrals over infinite intervals, an ability to find partial fraction decompositions, and remembering how to perform integration by parts. If any of these topics are unfamiliar to you, I strongly urge you to review them before this class meeting. You might also benefit from reading the first three pages of Section 6.1.

for Thursday, April 23

1. Read Section 6.1 of the text.
2. Do problems 1, 2, 5a, 9, 10, 14, 15, 21, 26, and 27 in Section 6.1. You will need to become familiar with functions such as those in problems 1 and 2 so don’t ignore them. For problem 5a, you will need to use integration by parts (and remember to use correct notation for improper integrals). For problems 9 and 10, you should use the linearity of the Laplace transform and previous results; there should be no integration. Problem 15 is another integration by parts problem, very similar to problem 5a. Finally, problem 26 introduces you to an important function, namely, the gamma function.
3. Turn in solutions to problems 12, 17, and 27a from Section 6.1. For problem 12, you should use the formula developed in class; pay particular attention to your notation for improper integrals. For problem 17, you should use the linearity of the Laplace transform and the result of problem 15.

for Friday, April 24

1. Read Section 6.2 of the text.
2. Do problems 1 through 12 in Section 6.2. You may decide not to do all of these problems; just be aware that doing these types of problems quickly and accurately will be important for this chapter.
3. Turn in solutions to the following two problems related to the ideas in Section 6.2.
   i. Find \( f(t) \) if the Laplace transform of \( f(t) \) is \( F(s) = \frac{4s + 7}{s^2 - 6s + 34} \).
   ii. Use the method of Laplace transforms to solve the initial value problem

\[
y'' - 2y' - 8y = 0, \quad y(0) = 0, \quad y'(0) = 1.
\]
for Tuesday, April 28

1. Read Section 6.2 again if necessary. Become very familiar with the table of Laplace transforms on page 317; you need to be able to find transforms and inverse transforms quickly.

2. Do problems 13, 14, 16, 22, 27, 28, 30, 33, and 37 in Section 6.2. Problems 28 and 37 may look scary, but they are not if you read carefully. Problems 30 and 33 then follow easily from problem 28. It is problem 27 that may require the most time. You will need to find some “new” Maclaurin series, particularly those for \( \arctan x \) and \((1 + x)^{-1/2}\). (These two series should be in your notes from Chapter 5.)

3. Turn in solutions for problems 23 and 27(c) from Section 6.2. For problem 23, which should be the IVP

\[
y'' + 2y' + y = 4e^{-t}, \quad y(0) = 2, \quad y'(0) = -1,
\]

after solving for \( Y(s) \), you can probably obtain the partial fraction decomposition using some creative but simple algebra; there is no need to put in constants \( A, B, \) and \( C \) and solve equations to find these values. However, you can always use this method as a default. For problem 27(c) (which should involve finding the Laplace transform of two variations on the Bessel function), the second part of the problem is much easier than the first so you might consider doing it first to see how the answer falls out. Then go through the following steps for the first part.

i. Find the Maclaurin series for \((1 + x)^{-1/2}\) by taking derivatives, finding a pattern, evaluating them at 0, then simplifying the coefficients. (As mentioned above, we did a variation on this problem in class two weeks ago. However, if you are still struggling with this sort of problem, going through the details is good practice.) Note that \( x = -1 \) is the nearest “bad” point so the radius of convergence of the series is 1. Replacing \( x \) with \( 1/s \) then gives a series that converges for \( s > 1 \).

ii. Assume that the Laplace transform of an infinite sum is just the sum of the Laplace transform of each term (this is a big assumption but it can be shown to be valid in this case) to find the Laplace transform for \( J_0(t) \). Linearity provides the key here so there is very little to do other than using appropriate formulas.

iii. With a little algebra, you should be able to match up the pieces from (i) and (ii).

4. Remember that one of the skills you can acquire through these problems is patience and attention to detail. This is a skill that is applicable in many areas of life and work. (Think about, for instance, a surgeon, a mechanic servicing an airplane, or someone testing the safety of a drug.)
for Thursday, April 30

1. Read Section 6.3 of the text.

2. Do problems 1, 2, 3, 5, 7, 8, 10, 13, 14, 15, 17, 20, 21, 23, 24, 25ab, 30, 31, 35, and 37 in Section 6.3. This is a lot of problems but most of them should go quickly.

3. Turn in solutions to problem 37 from Section 6.3 (note that you need to use the result found in problem 34) as well as solutions to the two additional problems given below.
   i. Find the Laplace transform of the function \( f \) defined by \( f(t) = u_1(t) t - u_4(t) (t - 2) \). In addition, sketch a graph of the function \( f \).
   ii. Find the inverse Laplace transform of the function \( F \) defined by \( F(s) = \frac{(s - 3)e^{-2s}}{s^2 + 2s + 10} \). 

for Friday, May 1

1. Read Section 6.4 of the text.

2. Do problems 1a, 3a, 5a, 9a, 13a, 14, and 15 in Section 6.4. For problem 9, show (algebraically) that the amplitude of the solution for \( t > 6 \) is \(|\cos 3|\). It would be a good idea to look at some of the graphs of the solutions for the first few problems using Maple or some other graphing device.

3. Turn in solutions for problem 6a in Section 6.4 and for the following problem:

   Solve the initial value problem \( y'' + y = f(t), \ y(0) = 0, \ y'(0) = 0 \), where

   \[
   f(t) = \begin{cases} 
   0, & \text{if } t < 0; \\
   t, & \text{if } 0 \leq t < 2; \\
   4 - t, & \text{if } 2 \leq t < 4; \\
   0, & \text{if } t \geq 4.
   \end{cases}
   \]

   After determining the solution, find the amplitude of the resulting harmonic motion for \( t \geq 4 \). If possible, find the exact value for this amplitude, otherwise (with the deduction of one homework point) express the answer to the nearest ten-thousandth.
for Tuesday, May 5

1. Read Section 6.5 of the text. It is rather short and spends most of its time explaining the Dirac delta function. Once you get the basic idea and are willing to believe that such a “function” can be used, the examples are rather easy computationally.
2. Do problems 1, 6, 7, 8, and 14ab in Section 6.5. For problem 7, ignore that strangely placed \( \cos t \).
3. Turn in solutions to the following two problems:
   
   i. Consider the initial value problem \( y'' + 2y' + 10y = k\delta(t - \pi), \ y(0) = 0, \ y'(0) = 0 \), where \( k \) is a positive constant to be determined. As you should be aware, the solution to this differential equation will be a decaying oscillation. Determine (to the nearest tenth) a value for \( k \) so that the maximum value of the solution is 4. Once you choose a value for \( k \), determine to the nearest ten-thousandth the maximum value (which will be close to 4 but not equal to 4 due to round-off error) and the minimum value of the corresponding solution. (This level of accuracy should force you to do some analysis, not just try to read numbers off of a graph.)
   
   ii. This problem is in the spirit of problem 6.2.27 (that is, I want you to learn this material). Find the Laplace transform for the function \( f \) defined by
      
      \[
      f(t) = \begin{cases} 
      (e^t - 1)/t, & \text{if } t \neq 0; \\
      1, & \text{if } t = 0; 
      \end{cases}
      \]

      by using the Maclaurin series of the function \( f \) and assuming that the linearity of the Laplace transform holds for infinite sums. You should then be able to identify as a “normal function” the function that you obtain. If you do not see it, replace \( 1/s \) with \( x \) and differentiate the power series of the transform. You should then recognize a familiar Maclaurin series and can convert it back to the function you need.
4. You may want to review what we have done thus far in Chapter 6 in preparation for the test on Friday. Make certain you are familiar with formulas for finding basic Laplace transforms (in both directions) as you will need to know them for the exam.

for Thursday, May 7

1. Read (or perhaps skim) Section 6.6 of the text.
2. Do problems 2, 3, 4, 8, 12, 15, and 17 in Section 6.6.
3. Turn in solutions for problem 3 and the following problem:
   
   i. Find \( t * t \) two ways; one using the definition and the other using Laplace transforms.
4. We will review for the exam in class on Thursday.
for Friday, May 8

1. We have a test on the topics we have covered in Chapter 6. You do need to be familiar with the table of Laplace transforms and know how to use the definition to derive the entries in the table.

2. Exams from previous courses on this material can be found on the website. At some point, you can use the following five problems as part of your review for the test. You should do these problems without the textbook or a calculator and keep track of how long they take (that is, mimic a testing situation). The answers are given below. Do note that correcting your solution after checking the answer indicates that you would have been incorrect on the exam. These problems are on the order of moderate difficulty for an exam.

i. Consider the initial value problem

\[ y'' + y = u_\pi(t) + 4\delta(t - 2\pi) - u_4\pi(t), \quad y(0) = 1, \quad y'(0) = 0. \]

Solve this initial value problem, then find the amplitude of the solution for \( t > 4\pi \).

ii. Find the Laplace transform of the function \( f \) defined by \( f(t) = u_4(t) t^2 \).

iii. Find the function \( f \) whose Laplace transform is given by \( F(s) = \frac{(s^2 - 3s + 1)e^{-2s}}{s(s^2 - 1)} \).

iv. Find the Laplace transform for the function \( f \) defined by

\[
 f(t) = \begin{cases} 
 (1 - \cos t)/t, & \text{if } t \neq 0; \\
 0, & \text{if } t = 0; 
\end{cases}
\]

and represent the transform in terms of familiar calculus functions.

v. Suppose that the Laplace transform of a function \( h(t) \) is represented by \( H(s) \). Given that \( h(0) = 1 \), \( h'(0) = 2 \), and \( h''(0) = 3 \), find (in terms of \( H \) ) the Laplace transform for the function \( \phi \) defined by

\[ \phi = h''' - 2h'' + 5h' - 4h. \]

i. The amplitude is 5.

ii. The Laplace transform \( F \) of \( f \) is defined by \( F(s) = \frac{2}{s^3}(8s^2 + 4s + 1)e^{-4s} \).

iii. The function \( f \) is given by \( f(t) = -\frac{1}{2} u_2(t)(2 + e^{t-2} - 5e^{2-t}) \).

iv. The Laplace transform \( F \) of \( f \) is defined by \( F(s) = \ln \frac{\sqrt{s^2 + 1}}{s} \).

v. The Laplace transform \( \Phi \) of \( \phi \) is defined by \( \Phi(s) = (s - 1)(s^2 - s + 4)H(s) - s^2 - 4 \).