Extra Practice: Algebra and Trig

1. Perform each of the following operations:
   (a) \[
   \frac{3s^2 - 48}{s^2 + 2s - 8} \div \frac{7s - 28}{s^2 - 4s + 4}
   \]
   (b) \[
   \frac{1}{p} + \frac{1}{q} \div \frac{1}{pq}
   \]

2. Simplify each of the following:
   (a) \[
   \frac{(p + 1)^{1/2} - p(1/2)(p + 1)^{-1/2}}{p + 1}
   \]
   (b) \[
   \frac{3(2x^2 + 5)^{1/3} - x(2x^2 + 5)^{-2/3}(4x)}{(2x^2 + 5)^{2/3}}
   \]
   (c) \[
   \frac{(r - 2)^{2/3} - r(2/3)(r - 2)^{-1/2}}{(r - 2)^{4/3}}
   \]

3. Practice with rules of exponents: Simplify, writing each expression without negative exponents:
   (a) \[
   \frac{6r^3s^{-2}}{6^{-1}r^4s^{-3}}
   \]
   (b) \[
   \frac{(2x^{-3})^2(3x^2)^{-2}}{6(x^2y^3)^{-1}}
   \]

4. Practice with the inverse trigonometric functions (See Section 1.6)
   (a) Find the exact value of each expression:
      \[
      \cos^{-1}(-1) \quad \arctan(1) \quad \sin^{-1}(1/\sqrt{2}) \quad \tan^{-1}(1/\sqrt{3})
      \]
   (b) Show (using a triangle) that \(\cos(\sin^{-1}(x)) = \sqrt{1 - x^2}\), then simplify \(\tan(\cos^{-1}(x))\) using a similar technique.

5. Solve for \(x\):
   (a) \(2\cos(x) + 1 > 0\) (in the interval \([0, 2\pi]\))
   (b) \((x - 2)/(x^2 - 2x - 3) \leq 0\)
   (c) \(\sin(x) > \cos(x)\) (in the interval \([0, 2\pi]\))
Solutions

1. Perform each of the following operations:
   (a) \( \frac{7}{3(s - 2)} \)
   (b) \( \frac{q + p}{pq - 1} \)

2. Simplify each of the following:
   (a) \( \frac{3p + 2}{2(p + 1)^{3/2}} \)
   (b) \( \frac{2x^2 + 15}{(2x^2 + 5)^{4/3}} \)
   (c) \( \frac{r - 6}{3(r - 2)^{5/3}} \)

3. Practice with rules of exponents: Simplify, writing each expression without negative exponents:
   (a) \( \frac{36s}{r} \)
   (b) \( \frac{2y^3}{27x^8} \)

4. Practice with the inverse trigonometric functions (See Section 1.6)
   (a) Find the exact value of each expression:
      \[
      \cos^{-1}(-1) = \pi \quad \arctan(1) = \frac{\pi}{4} \quad \sin^{-1}(1/\sqrt{2}) = \frac{\pi}{4} \quad \tan^{-1}(1/\sqrt{3}) = \frac{\pi}{6}
      \]
   (b) Show (using a triangle) that \( \cos(\sin^{-1}(x)) = \sqrt{1-x^2} \), then simplify \( \tan(\cos^{-1}(x)) \)
      using a similar technique.
      SOLUTION: For the first one, use \( \theta = \sin^{-1}(x) \), or \( \sin(\theta) = x \) and draw the appropriate right triangle. The hypotenuse is 1, the legs are \( x \) and \( \sqrt{1-x^2} \), so the cosine of the given angle is \( \sqrt{1-x^2}/1 \).
      Similarly, the right triangle for the second one label \( \theta \), \( x \) and 1 so that \( \cos(\theta) = x/1 \), then the other leg is \( \sqrt{1-x^2} \). Take the tangent of \( \theta \) to get \( \sqrt{1-x^2}/x \).

5. Solve for \( x \):
   (a) \( 2 \cos(x) + 1 > 0 \)
      SOLUTION: \( \cos(x) = -1/2 \) if \( x = 2\pi/3 \) or \( x = 4\pi/3 \) (on the unit circle). Between these angles, \( \cos(x) > -1/2 \), so: \( 0 < x < 2\pi/3 \) or \( 4\pi/3 < x < 2\pi \).
(b) \[ \frac{(x - 2)}{(x^2 - 2x - 3)} \leq 0 \] Use a sign chart. The factors are zero where \( x = -1, 2, \) and 3, which divides the number line into four parts. The ones in which the expression is negative are:

\[ x < -1 \quad \text{or} \quad 2 \leq x < 3 \]

(c) \( \sin(x) > \cos(x) \)

SOLUTION: Using \( \theta \), and the unit circle, we look at points \( x = \cos(\theta) \) and \( y = \sin(\theta) \) where \( y > x \). They are equal where the circle and the line \( y = x \) meet. At \( \theta = \pi/4 \) and \( 5\pi/4 \), and \( y > x \) in between:

\[ \frac{5\pi}{4} < \theta < \frac{\pi}{4} \]