Chapter 14

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Spring 2009
Definition

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**Example**

Find the domain: 

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$$\sqrt{y} \implies y \geq 0$$
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\[ \sqrt{y} \Rightarrow y \geq 0 \]

\[ 25 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 25 \]
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\[ f(x, y) = k \]

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Example: \( x^2 + 3xy - y^3 = 5 \)
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The graph of a function \( w = f(x, y, z) \) would have to have four dimensions (in order to plot four-tuples: \((x, y, z, w)\), and you could not visualize the same way we visualize other graphs.

We might think of time as one of the dimensions...
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To visualize three-dimensional graphs, look at the “level curves”: 

Definition

The **level curves** for a function $z = f(x, y)$ are curves where $k = f(x, y)$ (and note these are curves in the plane).
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Now a mathematical example:
Example: Plot the function \( (x^2 + 3y^2)e^{-x^2-y^2} \) by looking at it in 3-d and with the level curves: