\#4 Answer is 6
\#9 Hint: \(\int x \sin(x) \, dx\) - Integrate by parts.
\#12 Integrate with respect to \(x\) first; answer is \(\frac{1}{12}\).
\#14 Answer is \(\frac{3}{\pi}\).
\#15 Easiest as Type II; \(2 - y \leq x \leq 2y - 1, \ 1 \leq y \leq 2\).
\#20 Do as Type II; \(y^3 \leq x \leq y^2\) (Answer: \(16/21 \approx 0.08\)).
\#22 Not bad either as I or II; \(V = 5\pi\).
\#25 Type I: \(x^2 \leq y \leq 4, \ -2 \leq x \leq 2\).
\#26 (Started in class) Must do as
\[
\int_0^1 \int_{\sqrt{4-y^2}}^{2y} \sqrt{4-y^2} \, dx \, dy = \ldots = \frac{16}{3}
\]
\#31 In the \(xy\)-plane, we have \(y = 1-x^2\) and \(y = x^2 - 1\).

Above this region in the \(z\)-direction, the plane \(z = 2 - x + 2y + 10\) is above the plane \(z = 2 - x - y\).

We could compute the volume directly by taking the volume under the upper plane and subtracting the volume under the lower plane.
33. In the xy-plane, the region is:

\[ x + y = 1 \]

3d

39. \[ y = \sqrt{x} \]

40. \[ y = 4x \]

41. \[ x^2 + y^2 = 9 \]

42. \( y = 9 - x^2 \)

43. \( y = \ln(x) \) or \( x = e^y \)
\[ \int_0^4 \int_0^{y^2} \frac{1}{y^{2+1}} \, dx \, dy = \int_0^4 \frac{y^2}{y^{2+1}} \, dy \quad (u, \, du) \]

\[ \int_0^{\pi/2} \int_0^{\sqrt{\cos x}} \cos x \sqrt{1 + \cos^2 x} \, dy \, dx \]

\[ = \ldots = \int_0^{\pi/4} \cos x \sqrt{1 + \cos^2 x} \frac{\sin x \, dx}{\cos x} \]

\[ u = \cos x \]

\[ du = -\sin x \, dx \]

\[ \ldots \text{etc.} \]

51. (Sum 4 and 5 then)
52. (Break into 2)
53. (Look at Prop. 11, but you can skip this one).
55. The area of the triangle is 3/2, so the avg value:

\[ \text{avg } = \frac{1}{3/2} \int_0^1 \int_0^{3x} xy \, dy \, dx = \frac{2}{3} \int_0^1 \int_0^{3x} xy \, dy \, dx \ldots \]

61. See item 2 in the group work.