Note about Exercise 13, Section 10.2

There is some unusual algebra in Exercise 13. Given

\[ x = t - e^t \quad y = t + e^{-t} \]

we want to find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \). In class, we had:

\[
\frac{dx}{dt} = 1 - e^t \quad \frac{dy}{dt} = 1 - e^{-t} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1 - e^{-t}}{1 - e^t}
\]

Now the text says that \( \frac{dy}{dx} = -e^{-t} \). Where does that come from? (It is OK to leave your answer as the fraction by the way):

\[
\frac{dy}{dx} = \frac{1 - e^t}{1 - e^t} = \frac{e^t - 1}{e^t} \cdot \frac{1 - e^{-t}}{1 - e^{-t}} = -\frac{(1 - e^t)}{e^t} \cdot \frac{1}{1 - e^t} = -e^{-t}
\]

This makes the second derivative much easier to compute!

\[
\frac{d^2y}{dx^2} = \frac{d(-e^{-t})/dt}{dx/dt} = \frac{-e^{-t}}{1 - e^t}
\]

We could still get the answer from what we did in class, but it takes some work to get there. The numerator will be the derivative of \( \frac{dy}{dx} \):

\[
\frac{e^{-t}(1 - e^t) + (1 - e^{-t})e^t}{(1 - e^t)^2} = \frac{e^{-t} - 2 + e^t}{(1 - e^t)^2} = \frac{1}{e^t} - 2 + e^t = \frac{1 - 2e^t + e^{2t}}{1 - 2e^t + e^{2t}} = e^{-t}
\]

And the denominator is \( dx/dt \), or \( 1 - e^t \).

While the text’s answer is very nice, I wouldn’t expect you to simplify that far without knowing that it does simplify to something nice; but its good algebra practice to get the answer in the text.