Calculus III, Spring 2010
Solutions to the Quiz 1 Addendum:

1. (Same as earlier quiz set up) Let \( x = t - t^2 \) and \( y = \frac{4}{3}t^{3/2} \), \( 1 \leq t \leq 2 \). Set up the integral for the arc length and use Wolfram Alpha to give a numerical estimate:

   SOLUTION:
   \[
   \int_{1}^{2} \sqrt{1 + 4t^2} \, dt \approx 3.168
   \]

2. (Same as earlier quiz set up) Let \( x = 4t - t^3 \) and \( y = t^2 \). Get a parametric plot from Wolfram Alpha. Set up the integral for the area inside the loop, and evaluate it:

   SOLUTION: The right half of the loop is given for \( 0 \leq t \leq 2 \), so we have:
   \[
   2 \int_{0}^{2} x \, dy = 2 \int_{0}^{2} (4t - t^3) \, 2t \, dt = \int_{0}^{2} 8t^2 - 2t^4 \, dt = \frac{256}{15} \approx 17.06
   \]

3. Given the polar curve \( r = f(\theta) \), we can parameterize the it by using
   \[ x = r \cos(\theta) = f(\theta) \cos(\theta) \]
   \[ y = r \sin(\theta) = f(\theta) \sin(\theta) \]

   Find the equation of the tangent line (in Cartesian coordinates, \( (x, y) \)) for the polar function \( r = \cos(\theta) \) at \( \theta = \pi/4 \). For help with the derivative, see pages 644 and 645.

   SOLUTION: This was also done on Quiz 2. First, we take \( x, y \) to be functions of \( \theta \) and differentiate them:

   \[
   x = r \cos(\theta) = \cos^2(\theta) \quad \frac{dx}{d\theta} = -2 \cos(\theta) \sin(\theta) \quad \left. \frac{dx}{d\theta} \right|_{\theta=\pi/4} = -1
   \]

   \[
   y = r \sin(\theta) = \cos(\theta) \sin(\theta) \quad \frac{dy}{d\theta} = -\sin^2(\theta) + \cos^2(\theta) \quad \left. \frac{dy}{d\theta} \right|_{\theta=\pi/4} = 0
   \]

   Therefore, \( dy/dx = 0 \) at \( \theta = \pi/4 \). To find the equation, the point used (in Cartesian coordinates) is:

   \[
   x = \frac{1}{2} \quad y = \frac{1}{2}
   \]

   so the equation of the line is: \( y - \frac{1}{2} = 0(x - \frac{1}{2}) \), or simply \( y = 1/2 \).