To help you study, the following is basically last year’s exam (a couple of minor changes). Please write up answers (neatly!), and turn them in by Tuesday at the beginning of class. For the quiz (not on the exam), you may use a calculator, your text and your class notes to help you.

1. Short Answer:
   (a) What is the definition of the directional derivative?
   (b) Find the differential: \( w = xye^{xz} \)
   (c) If \( z = x^2y + 3xy^4 \) where \( x = \sin(2t) \) and \( y = \cos(2t) \), find \( dz/dt \) when \( t = 0 \).

2. Consider the function
   \[
   f(x, y) = \begin{cases}
   \frac{2x^2+3y^2}{x-y} & \text{if } x \neq y \\
   0 & \text{if } x = y
   \end{cases}
   \]
   Compute \( f_x(0, 0) \) by using the definition of the partial derivative.

3. Let \( f(x, y) = 1 + 2\sqrt{y} \). Find the rate of change of \( f \) at \((3, 4)\) in the direction of the vector \( \langle 4, -3 \rangle \).

4. Limits
   (a) Show that this limit exists at the origin by using the Squeeze Theorem:
   \[
   \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^2}
   \]
   (b) Show that this limit does not exist:
   \[
   \lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^2}
   \]
   (NOTE: On the exam, I would give you the figure. Since this is a take-home problem, you can plot it using Wolfram Alpha to help you).

5. Let \( f(x, y) = x^2 + y^2 - 2x - 4y \) be our height at \((x, y)\).
   (a) If we are at \((1, 1)\), in which direction should we move in order to move uphill the fastest? What is the rate of change if we move in that direction?
   (b) At \((1, 1)\), the gradient of \( f \) should be orthogonal to what level curve (and what kind of a curve is it?)
   (c) Find the equation of the tangent line and the normal line to the level curve at \((1, 1)\).
6. Suppose we have computed the directional derivative at \((1, 2)\) in the direction of the unit vector \(\langle u_1, u_2 \rangle\), and it was:

\[
D_u f(1, 2) = u_1 + u_2^2
\]

(a) Compute \(f_x(1, 2)\) and \(f_y(1, 2)\): 
(b) Is \(f\) differentiable at \((1, 1)\)? (Be specific)

7. If \(\vec{r}(t) = \langle \sin(t), \cos(t), t \rangle\)

(a) Find \(\int_0^\pi \vec{r}(t) \, dt\) 
(b) Find \(\mathbf{T}, \mathbf{N}\). 
(c) Find the arc length function \(s(t)\).

8. Find the maximum and minimum of \(f(x, y) = x^2y\) on the domain where \(x^2 + y^2 \leq 3\).

9. Find and classify the critical points using the second derivatives test:

\[
f(x, y) = x^3 - 6xy + 8y^3
\]