Exercise 34, 15.6

Rewrite
\[ \int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z) \, dy \, dz \, dx \]
in the order \( dx \, dy \, dz \).

First, draw a picture:

To re-do the integral, look at the projection of the solid into the \( yz \) plane:

The intersection of \( z = 1-x^2 \) with the plane \( y = 1-x \) projected into the \( yz \) plane:

This breaks the square into 2 parts. Points in region \( R_1 \):

Points in region \( R_2 \):

\[
\iint_{R_2} \int_0^{\sqrt{1-z}} f(x,y,z) \, dx \, dA + \iint_{R_1} \int_0^{1-y} f(x,y,z) \, dx \, dA
\]
To finish, express regions $R_2$ & $R_1$

\[ z = 1 - (1-y)^2 = 2y - y^2 \]

$R_2$: \[ 1 - (1-y)^2 \leq z \leq 1 \]
\[ 0 \leq y \leq 1 \]

$R_1$: \[ 0 \leq z \leq 1 - (1-y)^2 \]
\[ 0 \leq y \leq 1 \]

\[
\int_{y=0}^{1} \int_{z=0}^{\sqrt{1-z}} \int_{x=0}^{1-y} f(x, y, z) \, dx \, dz \, dy + \int_{y=0}^{1} \int_{z=0}^{2y-y^2} \int_{x=0}^{1-y} f(x, y, z) \, dx \, dz \, dy.
\]