Exam 1 Sample Questions

Be sure to look over your old quizzes and homework as well.

1. Eliminate the parameter to find a Cartesian equation for the curve: \( x = e^t, \ y = 2e^t \).

   Find the arc length of the curve from \( t = 1 \) to \( t = 2 \) of the original parametric form, then for the \((x, y)\) form.

2. Find \( dy/dx \) and \( d^2y/dx^2 \), if \( x = t^3 - 12t \) and \( y = t^2 - 1 \) (Hint: We do NOT want to try to convert it first).

3. Convert the polar equation to Cartesian:

   \[ r = \tan(\theta) \sec(\theta) \]

4. Convert the equation from Cartesian to polar of the form \( r = f(\theta) \).

   \[ xy = 4 \]

5. Find the area of the surface obtained by rotating the curve about the \( x-\)axis:

   \[ x = 3t - t^3 \quad y = 3t^2 \quad 0 \leq t \leq 1 \]

6. Find the slope of the tangent line to the given polar curve at the point specified by \( \theta \):

   \[ r = 2 - \sin(\theta) \quad \theta = \frac{\pi}{3} \]

7. Show that the equation \( r = a \sin(\theta) + b \cos(\theta) \), where \( ab \neq 0 \), represents the equation of a circle.

8. If we have two parallel planes, \( P_1 \) and \( P_2 \):

   \[ P_1 : \ ax + by + cz + d_1 = 0 \quad P_2 : \ ax + by + cz + d_2 = 0 \]

   Then show that the distance between the planes is

   \[ \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} \]

9. Graphical problems: Like the back of Quiz 3 (online), prob. 24, 25, 26 on p. 627.

10. Do the lines below intersect? If so, find the point of intersection. If not, find the distance between them.

    \[ x = 3 + t, \ y = 2 - t, \ z = 1 \quad x = 2 + s, \ y = 1 + 2s, \ z = 2 - s \]

11. Find an equation for the surface obtained by rotating the parabola \( y = x^2 \) about the \( y-\)axis. (Hint: In 3-d, if you fix a \( y-\)value, what shape should you have in the \( xz-\)plane?)
12. Find the distance between the origin and the line

\[ x = 1 + t \quad y = 2 - t \quad z = -1 + 2t \]

Hint: Take an arbitrary point on the line and form two vectors so that the distance can be found—perhaps with a sine? Hint 2: It is possible to solve this with Calculus—Use it to check your answer.

13. Given the property:

\[ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} \]

If vectors \( \mathbf{a} \) and \( \mathbf{b} \) are unit vectors, and

\[ \mathbf{c} = \mathbf{a} \times (\mathbf{a} \times \mathbf{b}) \]

- Is \( \mathbf{c} \) a unit vector?
- Is \( \mathbf{c} \perp \mathbf{a} \)?
- Is \( \mathbf{c} \perp \mathbf{b} \)?

14. Assume that \( \mathbf{a} \neq \mathbf{0} \). Explain your answer (if “No”, provide an example):

(a) If \( \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \), does it follow that \( \mathbf{b} = \mathbf{c} \)?
(b) If \( \mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \), does it follow that \( \mathbf{b} = \mathbf{c} \)?
(c) If \( \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \) and \( \mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c} \), does it follow that \( \mathbf{b} = \mathbf{c} \)?