Selected Solutions, Chapter 15 Review

T/F, 7 False. Notice that the \( r \) is missing: It should be \( r \, dz \, dr \, d\theta \)

10. \( \int_0^4 \int_{y-4}^{4-y} f(x, y) \, dx \, dy \)

12. The region is in the first octant on or between the spheres of radius 1 and radius 2.

14. \( \int_0^1 \int_0^{x^2} \frac{y e^{x^2}}{x^3} \, dy \, dx = \frac{1}{4}(e - 1) \)

18. Best to integrate in \( y \) first (otherwise you have to add two integrals together):

\[
\int_0^1 \int_x^1 \frac{1}{1 + x^2} \, dy \, dx = \int_0^1 \frac{1}{1 + x^2} - \frac{x}{1 + x^2} \, dx
\]

The first antiderivative is \( \tan^{-1}(x) \). To do the second, we can use \( u = 1 + x^2 \) and \( du = 2x \, dx \). Numerically, we get:

\[
\tan^{-1}(1) - \tan^{-1}(0) - \frac{1}{2} \ln(2) + \frac{1}{2} \ln(1) = \frac{\pi}{4} - \frac{1}{2} \ln(2)
\]

20. \( \int_1^2 \int_{1/y}^y y \, dx \, dy = \frac{4}{3} \)

26. (See figure) \( \int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{2-y} z \, dz \, dx \, dy = \frac{13}{24} \)

Figure 1: Figure for Exercise 26, Chapter 15 Review

30. \( \int_0^1 \int_{y+1}^{4-2y} \int_0^{x^2} z \, dz \, dx \, dy = \frac{53}{20} \)

46. See the figure below.
Selected Solutions, Chapter 16 Review

True/False (2) True, (4) True, (6) False (the arc length is independent of the parameterization), (8) False, since the divergence of the curl is not zero.

2. \( \frac{1}{12}(5\sqrt{5} - 1) \)

4. Conservative vector field, so 0.

   Side Remark: If you were curious, to get the parameterization re-write the ellipse in standard form:
   \[
   \frac{x^2}{9} + \frac{y^2}{4} = 1 \implies x(t) = 3 \cos(t) \quad y(t) = 2 \sin(t)
   \]

6. \( e - \frac{9}{70} \)

8. \( \pi/4 \)

10. In part (a), parameterize as \( x = 3 - 3t, y = \frac{\pi}{2} t, z = 3t \) for \( 0 \leq t \leq 1 \). With that, get \( \frac{1}{2}(3\pi - 9) \).

   In part (b), you should get \( -3\pi/4 \) (notice that this vector field is not path independent).

14. 2

16. 3

18. Messy, but straightforward:
   \[
   \langle -e^{-y} \cos(z), -e^{-z} \cos(x), -e^{-x} \cos(y) \rangle
   \]

   The divergence does not simplify too nicely, either:
   \[
   -e^{-x} \sin(y) - e^{-y} \sin(z) - e^{-z} \sin(x)
   \]

38. Deleted
Figure 2: Figure for Exercise 46, Chapter 15 Review