GROUP WORK 4, SECTION 12.3
Find the Error (Part 2)

After determining the stranger's mistake, you go to your calculus class. Your teacher tells you to pay particular attention to page 780, so you take a scrap of paper to mark the page. You notice that you are using the gentleman's receipt, and that he has written something on the back as well!

Dear Merry Grig,

If I haven't already convinced you that your teacher is nothing but a purveyor of falsity, check this out: Let \( \mathbf{u} \) be a vector such that \( |\mathbf{u}| = 1 \). Choose a vector \( \mathbf{v} \) such that \( \mathbf{u} \cdot \mathbf{v} = 3 \) and \( |\mathbf{v}| = \sqrt{5} \). Now we have

\[
|\mathbf{u} - \mathbf{v}|^2 = (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) \\
= \mathbf{u} \cdot \mathbf{u} - 2 (\mathbf{u} \cdot \mathbf{v}) + \mathbf{v} \cdot \mathbf{v} \\
= 0
\]

Hence \( \mathbf{u} - \mathbf{v} \), since \( \mathbf{u} - \mathbf{v} = 0 \). But \( \mathbf{u} \) and \( \mathbf{v} \) have different lengths!

Well, gosh darn him anyway! How can two things be the same, and yet different?
Find the error.